For simplicity, consider a real single with a two Fourier modes of frequency $\pm\omega_0$, and an arbitrary phase shift:

$$A(t) = \cos[\omega_0(t+\phi)] \tag{1}$$

Suppose one observes data over some finite time domain $-T \leq t \leq T$, where T is incommensurate with ω_0 (i.e., where $T\omega_0/2\pi$ is not integer). The periodic extension of $A(t)$ on this interval, which identifies the times $T = -T$, will have a *discontinuity* at this boundary. Fourier transformation of this incommensurate signal will therefore give rise to strong "ringing" artifacts in the power spectrum.

A suggestion to minimize these artifacts is to "symmetrize" the time signal:

$$\tilde{A}(t) \equiv \frac{A(t) + A(-t)}{2}. \tag{2}$$

Because $\tilde{A}(t) = \tilde{A}(-t)$, this symmetrized signal is no longer discontinuous at the periodic boundary $T = -T$. Instead, there is a weaker kink singularity, which should have less pronounced Gibbs ringing.

In Fourier space,

$$\begin{aligned}\tilde{A}_\omega &= \int e^{-i\omega t}\tilde{A}(t)dt \\ &= \operatorname{Re} A_\omega.\end{aligned} \tag{3}$$

The second step uses the assumption that the signal $A(t)$ is real.

Compute explicitly,

$$\begin{aligned}A_\omega &= \int e^{-i\omega t}\frac{e^{i\omega_0(t+\phi)} + e^{-i\omega_0(t+\phi)}}{2}dt \\ &\propto e^{i\omega_0\phi}\delta(\omega-\omega_0) + e^{-i\omega_0\phi}\delta(\omega+\omega_0)\end{aligned} \tag{4}$$

and

$$\tilde{A}_\omega = \operatorname{Re} A_\omega \propto \cos\omega_0\phi \left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]. \tag{5}$$

For $\omega_0 \neq 0$, we have

$$\delta(\omega-\omega_0)\delta(\omega+\omega_0) = 0, \tag{6}$$

such that the associated power spectra are

$$|A_\omega|^2 \propto \delta(\omega-\omega_0)^2 + \delta(\omega+\omega_0)^2. \tag{7}$$

$$|\tilde{A}_\omega|^2 \propto (\cos\omega_0\phi)^2 \left[\delta(\omega-\omega_0)^2 + \delta(\omega+\omega_0)^2\right]. \tag{8}$$

The prefactor is identical, so

$$|\tilde{A}_\omega|^2 = (\cos\omega_0\phi)^2\, |A_\omega|^2. \tag{9}$$

Observe the appearance of a nontrivial prefactor, $(\cos\omega_0\phi)^2$, which depends on the arbitrary phase shift. In practice, structure factor measurements involve equilibrium averages $\langle\cdot\rangle$, which will effectively average over all phases. This allows the effective replacement by the average prefactor,

$$(\cos\omega_0\phi)^2 \to \frac{1}{2\pi}\int_0^{2\pi}(\cos\theta)^2 d\theta = \frac{1}{2}, \tag{10}$$

such that

$$\left\langle |\sqrt{2}\tilde{A}_\omega|^2 \right\rangle = \langle |A_\omega|^2 \rangle$$

To summarize, the symmetrized signal

$$\frac{A(t)+A(-t)}{\sqrt{2}},$$

should give exactly the correct structure factor for the case of a single Fourier mode and a real signal.