Note

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Multi-Sensor Fusion: Covariance Intersection Using Tracks as Measurements

Background

The covariance intersection algorithm from Julier and Uhlmann [1] is a popular algorithm for track-to-track fusion in target tracking systems. This approach is highly appealing due to its robustness, simple structure, and applicability to any tracking system that uses Gaussians as the basis for tracking. Generalisations to non-Gaussian systems have been proposed based on the exponential mixture density structure of the algorithm. The approach is based on a simple rule called the Chernoff Fusion Rule. However, due to the non-Bayesian formulation of the rule, it cannot be integrated straightforwardly into multi-target tracking algorithms which are based on Bayesian formulations.

A new Bayesian formulation for covariance intersection was recently proposed which allows for the integration of the approach into multi-target tracking algorithms. [2] The new formulation recasts the fusion rule as a Bayesian update rule that calculates a normalisation constant which enables integration into different multi-target tracking algorithms.

In this example we demonstrate the approach with different multi-target trackers in a multi-platform scenario where the sensors output estimated target tracks instead of raw measurements. In real life situations, such sensors make multi-target tracking more accessible to new researchers because they don't have to know about or implement target filtering and/or tracking algorithms on their own. However, when there are multiple sensors measuring the same target space and they all produce estimated tracks, as demonstrated in this example, it is not immediately clear how to combine this information into a single set of tracks. This is where the method of covariance intersection comes in.

The concept of covariance intersection relies on the aforementioned Chernoff fusion rule [3] :

$$p_\omega(x_k) = \frac{p_1(x_k)^\omega p_2(x_k)^{1-\omega}}{\int p_1(x)^\omega p_2(x)^{1-\omega} dx}$$

In situations where $p_1(x)$ and $p_2(x)$ are multivariate Gaussian distributions, this formula is equal to the Covariance Intersection Algorithm from Julier and Uhlmann. In the Covariance Intersection Algorithm, the weighting parameter, $\omega \in [0,1]$ is chosen using an optimization algorithm. In this example, we have set it to 0.5 for simplicity.

We also introduce the following identity. Given two Gaussians, $N(x;a,A)$ and $N(x;b,B)$ with the same dimension, we have:

$$\left(\frac{\mathcal{N}(x;a,A)}{\mathcal{N}(x;a,A)}\right)^\omega \mathcal{N}(x;b,B) \propto \mathcal{N}(a;b,V)\mathcal{N}(x;d,D)$$

where

$$\begin{aligned} D &= \left(\omega A^{-1} + (1-\omega)B^{-1}\right)^{-1} \\ d &= D\left(\omega A^{-1}a + (1-\omega)B^{-1}b\right) \\ V &= A/(1-\omega) + B/\omega \end{aligned}$$

This example considers the Gaussian mixture probability hypothesis density (GM-PHD) algorithm as the tracker for the track-to-track fusion. The following table shows the formulas used in the regular GM-PHD, and the GM-PHD covariance intersector algorithm.

	Traditional GM-PHD	**GM-PHD Covariance Intersector**
Posterior/Fused Intensity	$v_k(x) = (1 - p_{D,k})v_{k\|k-1}(x) + \sum_{z \in Z_k} v_{D,k}(x; z)$	$v_k(x) = (1 - p_E)v_{k\|k-1}(x) + \sum_{(a_k, A_k) \in \mathcal{A}_k} v_{E,k}(x; (a_k, A_k))$
Individual Gaussian component	$v_{D,k}(x; z) = \sum_{j=1}^{J_{k\|k-1}} w_k^{(j)}(z)\mathcal{N}\left(x; m_{k\|k}^{(j)}(z), P_{k\|k}^{(j)}\right)$	$v_{E,k}(x; (a_k, A_k)) = \sum_{j=1}^{J_{k\|k-1}} w_k^{(j)}(a_k, A_k)\mathcal{N}\left(x; c_{k\|k}^{(j)}(a_k, A_k), C_{k\|k}^{(j)}\right)$
Updated weight	$w_k^{(j)}z = \frac{p_D w_{k\|k-1}^{(j)} q_k^{(j)}(z)}{\kappa_k(z) + p_{D,k}\sum_{l=1}^{J_{k\|k-1}} w_{k\|k-1}^{(j)} q_k^{(l)}(z)}$	$w_k^{(j)}(a_k, A_k) = \frac{p_E w_{k\|k-1}^{(j)} u_k^{(j)}(a_k, A_k)}{\lambda + p_E \sum_{l=1}^{J_{k\|k-1}} u_k^{(l)}(a_k, A_k)}$
Innovation	$q_k^{(j)}(z) = \mathcal{N}\left(z;\ H_k m_{k\|k-1}^{(j)}, S_k^{(j)}\right)$	$u_k^{(j)}(a_k, A_k) = \mathcal{N}\left(a_k; m_{k\|k-1}^{(j)}, V_k^{(j)}\right)$
Updated mean	$m_{k\|k}^{(j)}(z) = m_{k\|k-1}^{(j)} + K_k^{(j)}(z - H_k m_{k\|k-1}^{(j)})$	$c_{k\|k}^{(j)}(a_k, A_k) = C_{k\|k}^{(j)}\left(\omega\left(P_{k\|k-1}^{(j)}\right)^{-1} m_{k\|k-1}^{(j)} + (1-\omega){A_k}^{-1}a_k\right)$
Updated covariance	$P_{k\|k}^{(j)} = \left[I - K_k^{(j)}H_k\right]P_{k\|k-1}^{(j)}$	$C_{k\|k}^{(j)} = \left(\omega\left(P_{k\|k-1}^{(j)}\right)^{-1} + (1-\omega){A_k}^{-1}\right)^{-1}$
Kalman gain	$K_k^{(j)} = P_{k\|k-1}^{(j)}{H_k}^T\left(S_k^{(j)}\right)^{-1}$	
Innovation covariance	$S_k^{(j)} = R_k + H_k P_{k\|k-1}^{(j)}{H_k}^T$	$V_k^{(j)} = \frac{P_{k\|k-1}^{(j)}}{\omega} + \frac{A_k}{1-\omega}$
	Z_k – random finite set of measurements p_D – probability of detection κ – clutter intensity H – measurement matrix R – measurement noise covariance I – Identity matrix	$\mathcal{A}_k$– set of tracks, consisting of (mean, covariance) pairs (a_k, A_k) p_E – probability that any given target has been estimated in tracks $\mathcal{A}_k$ by the sensor λ – intensity of false tracks ω – the weighting parameter in range $(0, 1]$

The rest of this example will continue as follows:

- Create a simulator for the ground truth
- Create 2 radar simulators, one on the ground and one that is airborne
- Make a JPDA tracker for the first radar, and a Gaussian mixture linear complexity with cumulants (GM-LCC) tracker for the second. These will mimic the situation where the radar sensors outputs tracks instead of raw measurements.
- Create a GM-PHD tracker that will perform measurement fusion, using all measurements from both radars. This is created to compare with the covariance intersection method.
- Define the ChernoffUpdater class, which implements the covariance intersection
- Define a Detector class which converts tracks into Gaussian measurements
- Create a GM-PHD tracker using the ChernoffUpdater for the covariance intersection
- Create metric managers for each of the four trackers
- Run the simulation, plot the resulting tracks, and plot the metrics over time

```
from copy import deepcopy
import numpy as np
from datetime import datetime

start_time = datetime.now()
num_steps = 50
```

1: Create a Ground Truth Simulator

We will simulate the paths of two targets using the `MultiTargetGroundTruthSimulator`. We can dictate the starting states of the two targets using the *preexisting_states* parameter. The targets start at [-100, -200, 500] and [0, 300, 500] respectively. Their initial velocities are [4, 0.5, 0] and [5, -0.5, 0] and they move according to a constant velocity transition model with noise.

```
from stonesoup.models.transition.linear import CombinedLinearGaussianTransitionModel,\
                                               ConstantVelocity
truth_transition_model = CombinedLinearGaussianTransitionModel(
    (ConstantVelocity(0.5), ConstantVelocity(0.5), ConstantVelocity(0.5)))

from stonesoup.simulator.simple import MultiTargetGroundTruthSimulator
from stonesoup.types.state import GaussianState
gt_simulator = MultiTargetGroundTruthSimulator(
    transition_model=truth_transition_model,
    initial_state=GaussianState([0, 0, 0, 0, 500, 0], np.diag([100, 1, 100, 1, 100, 1]),
                                timestamp=start_time),
    birth_rate=0,
    death_probability=0,
    number_steps=num_steps,
    preexisting_states=[[-100, 4, -200, 0.5, 500, 0], [0, 5, 300, -0.5, 500, 0]]
)
```

2: Create Two Radars and a Detection Simulation

The two radars can share the same clutter model.

```
from stonesoup.models.clutter.clutter import ClutterModel
clutter_model = ClutterModel(
    clutter_rate=2.0,
    distribution=np.random.default_rng().uniform,
    dist_params=((-600.0, 600.0), (-600.0, 600.0), (250.0, 750.0))
)
```

The first radar will be airborne, at an altitude of approximately 3000 m. It makes detections with an elevation, bearing, and range measurement model. By setting the max_range to 3500, we can ensure that it does not make detections of the other radar (which will be far away on the ground). We will later do a similar thing with the second radar. This mimics a real-life scenario where each radar is outside the field-of-view of the other.

```
from stonesoup.sensor.radar.radar import RadarElevationBearingRange
from stonesoup.types.array import CovarianceMatrix
from stonesoup.types.array import StateVector
from stonesoup.platform.base import MovingPlatform
from stonesoup.types.state import State

radar1 = RadarElevationBearingRange(
    ndim_state=6,
    position_mapping=(0, 2, 4),
    noise_covar=CovarianceMatrix(np.diag([np.deg2rad(0.005), np.deg2rad(0.005), 0.05])),
    mounting_offset=StateVector([10, 0, 0]),
    clutter_model=clutter_model,
    max_range=3500
)

# Mount the radar onto a moving platform. The platform starts at [-250, 50, 3000]
# with velocity [1, 5, 0] and moves according to a constant velocity model with low noise
sensor1_initial_loc = StateVector([[-250], [1], [50], [5], [3000], [0]])
initial_state = State(sensor1_initial_loc, start_time)
sensor1_transition_model = CombinedLinearGaussianTransitionModel(
    [ConstantVelocity(0.3), ConstantVelocity(0.3), ConstantVelocity(0.3)])
sensor1_platform = MovingPlatform(
    states=initial_state,
    position_mapping=(0, 2, 4),
    velocity_mapping=(1, 3, 5),
    transition_model=sensor1_transition_model,
    sensors=[radar1]
)
```

The second radar will be stationary on the ground at the point [2000, 50, 0]. This radar also measures in 3D using bearing, range, and elevation.

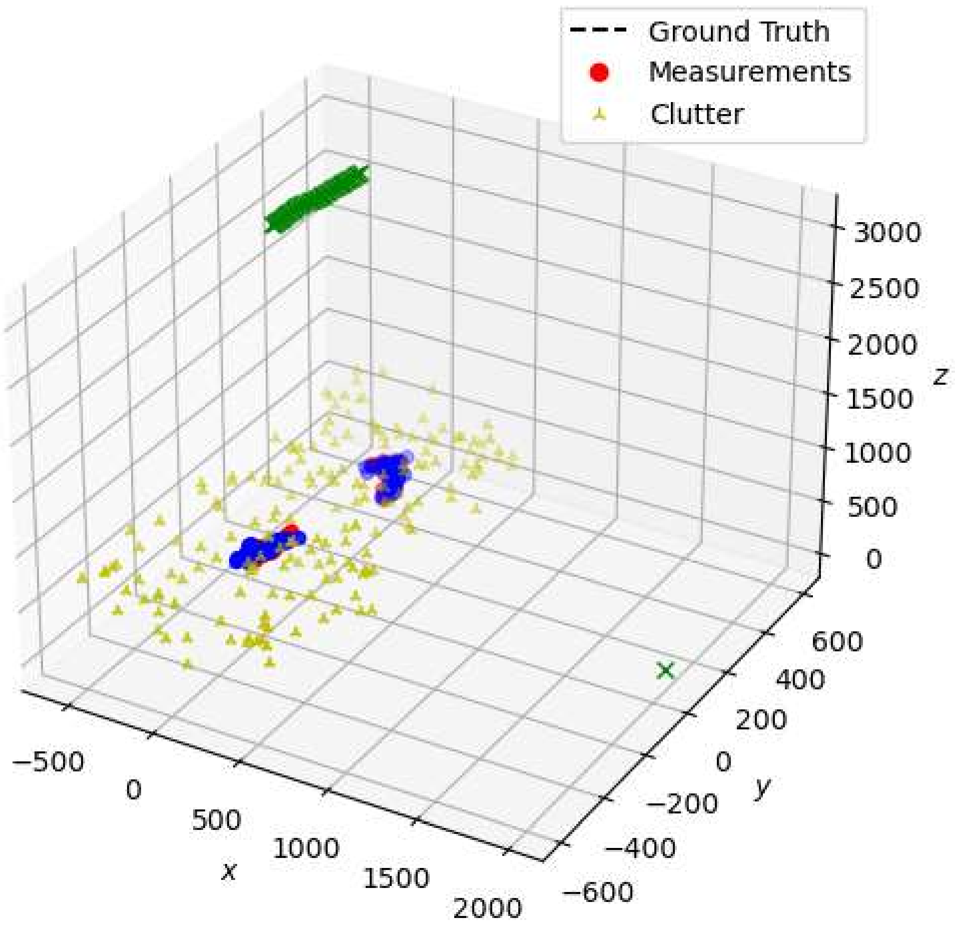
```
radar2_noise_covar = CovarianceMatrix(np.diag([np.deg2rad(0.005), np.deg2rad(0.005), 0.05]))
radar2 = RadarElevationBearingRange(
    ndim_state=6,
    position_mapping=(0, 2, 4),
    noise_covar=radar2_noise_covar,
    clutter_model=clutter_model,
    max_range=3000
)

# Make a platform and mount the radar
from stonesoup.platform.base import FixedPlatform
sensor2_platform = FixedPlatform(
    State([2000, 0, 50, 0, 0, 0]),
    position_mapping=[0, 2, 4],
    sensors=[radar2]
)
```

Now we can pass the platforms into a detection simulator. At each timestep, the simulator will return the detections from the sensor1 platform, then the detections from the sensor2 platform.

```
from stonesoup.simulator.platform import PlatformDetectionSimulator
radar_simulator = PlatformDetectionSimulator(
    groundtruth=gt_simulator,
    platforms=[sensor1_platform, sensor2_platform]
)
```

Let’s briefly visualize the truths and measurements before we move on. Note that the final simulation will not have the same truths because the ground truth generator is randomized. But this gives an idea of what it will look like. The detections from the first sensor (airborne) will be plotted in blue, and the detections from the second sensor are in red. The clutter from both sensors are plotted in yellow. The sensor locations will be plotted in green Xs.

```
from stonesoup.plotter import Plotter, Dimension

# Lists to hold the detections from each sensor and the path of the airborne radar
s1_detections = []
s2_detections = []
radar1_path = []

# Extract the generator function from a copy of the simulator
sim = deepcopy(radar_simulator)
g = sim.detections_gen()

# Iterate over the time steps, extracting the detections, truths, and airborne sensor path
for _ in range(num_steps):
    s1_detections.append(next(g)[1])
    s2_detections.append(next(g)[1])
    radar1_path.append(sim.platforms[0].position)
truths = set(sim.groundtruth.groundtruth_paths)

# Plot the truths and detections
plotter = Plotter(dimension=Dimension.THREE)
plotter.plot_ground_truths(truths, [0, 2, 4])
plotter.plot_measurements(s1_detections, [0, 2, 4], color='blue')
plotter.plot_measurements(s2_detections, [0, 2, 4], color='red')

# Plot the radar positions
plotter.ax.plot(*zip(*radar1_path), marker='x', color='green')
plotter.ax.plot(2000, 50, 0, marker='x', color='green')
```



Out:

```
[<mpl_toolkits.mplot3d.art3d.Line3D object at 0x000002A7656A4130>]
```

3: Make Trackers for the Radars

The airborne radar will be tracking using a JPDA tracker, and the stationary one will use a GM-LCC. These trackers will not be given the platform detection simulation objects as parameters, we will feed the measurements later to ensure that that the same measurements are used in the fusion trackers. To start, we can calculate the clutter spatial density.

```
clutter_area = np.prod(np.diff(clutter_model.dist_params))
clutter_spatial_density = clutter_model.clutter_rate/clutter_area
```

JPDA Tracker

```
from stonesoup.hypothesiser.probability import PDAHypothesiser
from stonesoup.updater.kalman import ExtendedKalmanUpdater
from stonesoup.predictor.kalman import ExtendedKalmanPredictor
from stonesoup.dataassociator.probability import JPDA
from stonesoup.deleter.error import CovarianceBasedDeleter
from stonesoup.initiator.simple import MultiMeasurementInitiator
from stonesoup.tracker.simple import MultiTargetMixtureTracker

# Updater
jpda_updater = ExtendedKalmanUpdater(measurement_model=None)

# Data Associator
predictor = ExtendedKalmanPredictor(truth_transition_model)
hypothesiser = PDAHypothesiser(
    predictor=predictor,
    updater=jpda_updater,
    clutter_spatial_density=clutter_spatial_density,
    prob_detect=0.9
)
data_associator = JPDA(hypothesiser=hypothesiser)

# Deleter
covariance_limit_for_delete = 500
deleter = CovarianceBasedDeleter(covar_trace_thresh=covariance_limit_for_delete)

# Initiator
s_prior_state = GaussianState([0, 0, 0, 0, 500, 0], np.diag([0, 50, 0, 50, 0, 50]))
from stonesoup.hypothesiser.distance import DistanceHypothesiser
from stonesoup.measures import Mahalanobis
hypothesiser = DistanceHypothesiser(
    predictor,
    jpda_updater,
    measure=Mahalanobis(),
    missed_distance=3
)

from stonesoup.dataassociator.neighbour import GNNWith2DAssignment
initiator_associator = GNNWith2DAssignment(hypothesiser)
initiator_deleter = CovarianceBasedDeleter(covar_trace_thresh=500)
initiator = MultiMeasurementInitiator(
    prior_state=s_prior_state,
    measurement_model=None,
    deleter=initiator_deleter,
    data_associator=initiator_associator,
    updater=jpda_updater,
    min_points=2
)

jpda_tracker = MultiTargetMixtureTracker(
    initiator=initiator,
    deleter=deleter,
    detector=None,
    data_associator=data_associator,
    updater=jpda_updater
)
```

GM-LCC Tracker

```python
from stonesoup.updater.pointprocess import LCCUpdater
from stonesoup.hypothesiser.distance import DistanceHypothesiser
from stonesoup.measures import Mahalanobis
from stonesoup.hypothesiser.gaussianmixture import GaussianMixtureHypothesiser
from stonesoup.mixturereducer.gaussianmixture import GaussianMixtureReducer
from stonesoup.types.state import TaggedWeightedGaussianState
from stonesoup.tracker.pointprocess import PointProcessMultiTargetTracker

# Updater
kalman_updater = ExtendedKalmanUpdater(measurement_model=None)
updater = LCCUpdater(
    updater=kalman_updater,
    clutter_spatial_density=clutter_spatial_density,
    normalisation=True,
    prob_detection=0.9,
    prob_survival=0.9,
    mean_number_of_false_alarms=clutter_model.clutter_rate,
    variance_of_false_alarms=100
)

# Hypothesiser
kalman_predictor = ExtendedKalmanPredictor(truth_transition_model)
base_hypothesiser = DistanceHypothesiser(
    predictor=kalman_predictor,
    updater=kalman_updater,
    measure=Mahalanobis(),
    missed_distance=15,
    include_all=False
)
hypothesiser = GaussianMixtureHypothesiser(
    base_hypothesiser,
    order_by_detection=True
)

# Reducer
reducer = GaussianMixtureReducer(
    prune_threshold=1E-3,
    pruning=True,
    merge_threshold=200,
    merging=True
)

# Birth component
birth_covar = CovarianceMatrix(np.diag([10000, 10, 10000, 10, 10000, 10]))
birth_component = TaggedWeightedGaussianState(
    state_vector=[0, 0, 0, 0, 500, 0],
    covar=birth_covar**2,
    weight=0.5,
    tag=TaggedWeightedGaussianState.BIRTH,
    timestamp=start_time
)

# Tracker
gmlcc_tracker = PointProcessMultiTargetTracker(
    detector=None,
    hypothesiser=deepcopy(hypothesiser),
    updater=deepcopy(updater),
    reducer=deepcopy(reducer),
    birth_component=deepcopy(birth_component),
    extraction_threshold=0.90,
)
```

4: Make GM-PHD Tracker For Measurement Fusion

This tracker can use many of the same elements as the GM-LCC one.

```
from stonesoup.updater.pointprocess import PHDUpdater

updater = PHDUpdater(
    kalman_updater,
    clutter_spatial_density=clutter_spatial_density,
    prob_detection=0.9,
    prob_survival=0.9
)

meas_fusion_tracker = PointProcessMultiTargetTracker(
    detector=None,
    hypothesiser=deepcopy(hypothesiser),
    updater=deepcopy(updater),
    reducer=deepcopy(reducer),
    birth_component=deepcopy(birth_component),
    extraction_threshold=0.90,
)
```

5: Define the Chernoff Updater Class

The ChernoffUpdater is where we will implement the covariance intersection. As an `Updater`, the ChernoffUpdater will need functions *predict_measurement()* and *update()*.

The value of *omega* in the covariance intersection method is adjustable. We will set it to 0.5 for now.

```python
from stonesoup.updater.base import Updater
from stonesoup.types.prediction import GaussianMeasurementPrediction
from stonesoup.types.update import GaussianStateUpdate


class ChernoffUpdater(Updater):

    def predict_measurement(self, predicted_state, measurement_model=None,  **kwargs):
        '''
        This function modifies the usual hypothesis.measurement to use a state covariance
        and state vector (ie a track) as an input.
        '''

        # The innovation covariance uses the noise covariance from the measurement model
        state_covar_m = measurement_model.noise_covar
        omega = 0.5
        innov_cov = 1/(1-omega)*state_covar_m+1/omega*predicted_state.covar

        # The predicted measurement and measurement cross covariance can be taken from
        # the predicted state
        pred_meas = predicted_state.state_vector
        meas_cross_cov = predicted_state.covar

        # Combine everything into a GaussianMeasurementPrediction object
        return GaussianMeasurementPrediction(pred_meas, innov_cov,
                                             predicted_state.timestamp,
                                             cross_covar=meas_cross_cov)

    def update(self, hypothesis, force_symmetric_covariance=False, **kwargs):
        '''
        Given a hypothesis, calculate the posterior mean and covariance
        '''

        # Get the predicted state out of the hypothesis
        predicted_state = hypothesis.prediction

        # Extract the vector and covariance from the measurement
        state_covar_m = hypothesis.measurement.covar
        state_vector_m = hypothesis.measurement.state_vector

        # Predict the measurement if it is not already done
        if hypothesis.measurement_prediction is None:
            measurement_model = self._check_measurement_model(
                hypothesis.measurement.measurement_model
            )
            hypothesis.measurement_prediction = self.predict_measurement(
                predicted_state,
                measurement_model=measurement_model,
                **kwargs
            )

        # Calculate the updated mean and covariance from covariance intersection
        omega = 0.5
        posterior_covariance = np.linalg.inv(omega*np.linalg.inv(state_covar_m)
                                             + (1-omega)*np.linalg.inv(predicted_state.covar))
        posterior_mean = posterior_covariance@(omega*np.linalg.inv(state_covar_m)@state_vector_m
                                               + (1-omega)*np.linalg.inv(predicted_state.covar)
                                               @ predicted_state.state_vector)
        # Optionally force the posterior covariance to be a symmetric matrix
        if force_symmetric_covariance:
            posterior_covariance = \
                (posterior_covariance + posterior_covariance.T)/2

        # Return the updated state
        return GaussianStateUpdate(posterior_mean, posterior_covariance,
                                   hypothesis,
                                   hypothesis.measurement.timestamp)
```

6: Define a Detector Class Using Tracks as Measurements

In Stone Soup, normal measurements are fed into a tracker using a `DetectionFeeder` object. But the idea of covariance intersection is to use tracks (and thus, states) instead of raw measurements. Thus, we have to make a custom `DetectionFeeder` class which will work with tracks. This custom class will accept a collection of live tracks and take the most recent state from each track. Each of those states is then turned into a `GaussianDetection` object. All of the converted detections are returned and can be fed into a regular GM-PHD tracker.

```
from stonesoup.types.detection import GaussianDetection
from stonesoup.feeder.base import DetectionFeeder
from stonesoup.models.measurement.linear import LinearGaussian


class Tracks2GaussianDetectionFeeder(DetectionFeeder):
    '''
    A class which converts tracks to GaussianDetection objects.

    At each time step, look at the set of live tracks being fed in. Take the
    most recent state from each of those tracks, and turn it into a
    GaussianDetection object. The covariance of the measurement model will
    be equal to the state covariance.
    '''
    def data_gen(self):
        for time, tracks in self.reader:
            detections = []
            for track in tracks:
                detections.append(
                    GaussianDetection.from_state(
                        track.state,
                        measurement_model=LinearGaussian(6, [0, 1, 2, 3, 4, 5],
                                                         np.asarray(track.covar)),
                        target_type=GaussianDetection)
                )
            yield time, detections
```

7: Define a GM-PHD Tracker for Track Fusion

The tracker parameters have been kept the same as the measurement fusion tracker except where noted. This will ensure a fair comparison of the results.

```python
from stonesoup.measures import Euclidean

# Updater
# Here we insert the ChernoffUpdater as the base updater, instead of a typical
# :class:`~.KalmanUpdater`. The `clutter_spatial_density` parameter now refers to the estimated
# intensity of false tracks. Since the previous trackers will (hopefully) have ignored some of the
# clutter, we can user a smaller intensity than the others
ch_updater = ChernoffUpdater(measurement_model=None)
updater = PHDUpdater(
    ch_updater,
    clutter_spatial_density=1E-15,
    prob_detection=0.9,
    prob_survival=0.9
)


# Hypothesiser
# The states being used as measurements are in Cartesian space. We will use Euclidean distance in
# the :class:`~.DistanceHypothesiser`, meaning that we need a bigger missed distance than the
# previous hypothesiser which used the Mahalanobis distance.
kalman_predictor = ExtendedKalmanPredictor(truth_transition_model)
base_hypothesiser = DistanceHypothesiser(
    kalman_predictor,
    ch_updater,
    Euclidean(),
    missed_distance=300,
    include_all=False
)
hypothesiser = GaussianMixtureHypothesiser(base_hypothesiser, order_by_detection=True)

# Reducer
# The states tend to have low weights when they are first initialized using this method, so we
will
# keep the pruning threshold low.
ch_reducer = GaussianMixtureReducer(
    prune_threshold=1E-9,
    pruning=True,
    merge_threshold=200,
    merging=True
)

# Birth component
birth_covar = CovarianceMatrix(np.diag([100000, 100, 100000, 100, 100000, 100]))
ch_birth_component = TaggedWeightedGaussianState(
    state_vector=[0, 0, 0, 0, 500, 0],
    covar=birth_covar**2,
    weight=0.5,
    tag=TaggedWeightedGaussianState.BIRTH,
    timestamp=start_time
)

# Make tracker
track_fusion_tracker = PointProcessMultiTargetTracker(
    detector=None,
    hypothesiser=hypothesiser,
    updater=updater,
    reducer=deepcopy(ch_reducer),
    birth_component=deepcopy(ch_birth_component),
    extraction_threshold=0.90,
)
```

8: Make Metric Managers

We will track metrics for each of the four trackers for comparison.

```
from stonesoup.metricgenerator.basicmetrics import BasicMetrics
from stonesoup.metricgenerator.ospametric import OSPAMetric
from stonesoup.metricgenerator.tracktotruthmetrics import SIAPMetrics
from stonesoup.metricgenerator.uncertaintymetric import SumofCovarianceNormsMetric
from stonesoup.dataassociator.tracktotrack import TrackToTruth
from stonesoup.metricgenerator.manager import SimpleManager

# Make the basic metric manager
basic_generator = BasicMetrics()
ospa_generator = OSPAMetric(c=10, p=1, measure=Euclidean([0, 2, 4]))
siap_generator = SIAPMetrics(position_measure=Euclidean(), velocity_measure=Euclidean())
uncertainty_generator = SumofCovarianceNormsMetric()

associator = TrackToTruth(association_threshold=30)
base_metric_manager = SimpleManager([basic_generator, ospa_generator, siap_generator,
                                     uncertainty_generator],
                                    associator=associator)

sensor1_mm, sensor2_mm = deepcopy(base_metric_manager), deepcopy(base_metric_manager)
meas_fusion_mm, track_fusion_mm = deepcopy(base_metric_manager), deepcopy(base_metric_manager)
```

9: Run Simulation

As one final step before running the simulation, we will write a little class which feeds the detections for a single timestep. This makes sure that the two radars and the measurement fusion tracker are getting the same measurements.

```
from stonesoup.buffered_generator import BufferedGenerator
from stonesoup.reader.base import DetectionReader


class DummyDetector(DetectionReader):
    def __init__(self, *args, **kwargs):
        self.current = kwargs['current']

    @BufferedGenerator.generator_method
    def detections_gen(self):
        yield self.current
```

Now we are ready to run the simulation.

```
sensor1_detections, sensor2_detections = [], []
jpda_tracks, gmlcc_tracks = set(), set()
meas_fusion_tracks, track_fusion_tracks = set(), set()

sim_generator = radar_simulator.detections_gen()

for t in range(num_steps):

    # Run JPDA tracker from sensor 1
    s1d = next(sim_generator)
    sensor1_detections.extend(s1d[1])  # hold in list for plotting
    # Pass the detections into a DummyDetector and set it up as an iterable
    jpda_tracker.detector = DummyDetector(current=s1d)
    jpda_tracker.__iter__()
    # Run the tracker and store the resulting tracks
    _, sensor1_tracks = next(jpda_tracker)
    jpda_tracks.update(sensor1_tracks)

    # Run GM-LCC tracker from sensor 2
    s2d = next(sim_generator)
    sensor2_detections.extend(s2d[1])  # hold in list for plotting
    # Pass the detections into a DummyDetector and set it up as an iterable
    gmlcc_tracker.detector = DummyDetector(current=s2d)
    gmlcc_tracker.__iter__()
    # Run the tracker and store results
    time, sensor2_tracks = next(gmlcc_tracker)
    gmlcc_tracks.update(sensor2_tracks)

    # Run the GM-PHD for measurement fusion. This one gets called twice, once for each set of
    # detections. This ensures there is only one detection per target.
    for detections in [s1d, s2d]:
        meas_fusion_tracker.detector = DummyDetector(current=detections)
        meas_fusion_tracker.__iter__()
        _, tracks = next(meas_fusion_tracker)
        meas_fusion_tracks.update(tracks)

    # Run the GM-PHD for track fusion. Similar to the measurement fusion, this tracker gets run
    # twice, once for each set of tracks.
    for tracks_as_meas in [sensor1_tracks, sensor2_tracks]:
        dummy_detector = DummyDetector(current=[time, tracks_as_meas])
        track_fusion_tracker.detector = Tracks2GaussianDetectionFeeder(dummy_detector)
        track_fusion_tracker.__iter__()
        _, tracks = next(track_fusion_tracker)
        track_fusion_tracks.update(tracks)

    # ----------------------------------------------------------------------

    # Add ground truth data to metric managers
    truths = radar_simulator.groundtruth.current
    for manager in [sensor1_mm, sensor2_mm, meas_fusion_mm, track_fusion_mm]:
        manager.add_data(groundtruth_paths=truths[1], overwrite=False)

    # Add measurements to metric managers
    sensor1_mm.add_data(detections=s1d[1], overwrite=False)
    sensor2_mm.add_data(detections=s2d[1], overwrite=False)
    meas_fusion_mm.add_data(detections=s1d[1], overwrite=False)
    meas_fusion_mm.add_data(detections=s2d[1], overwrite=False)
    track_fusion_mm.add_data(detections=s1d[1], overwrite=False)
    track_fusion_mm.add_data(detections=s2d[1], overwrite=False)
```

```
# Ensure that all tracks have been extracted from the trackers
jpda_tracks.update(jpda_tracker.tracks)
gmlcc_tracks.update(gmlcc_tracker.tracks)
meas_fusion_tracks.update(meas_fusion_tracker.tracks)
track_fusion_tracks.update(track_fusion_tracker.tracks)

# Remove tracks that have just one state in them as they were probably from clutter
jpda_tracks = set([track for track in jpda_tracks if len(track) > 1])
gmlcc_tracks = set([track for track in gmlcc_tracks if len(track) > 1])
meas_fusion_tracks = set([track for track in meas_fusion_tracks if len(track) > 1])
track_fusion_tracks = set([track for track in track_fusion_tracks if len(track) > 1])

# Add tracks to metric managers
sensor1_mm.add_data(tracks=jpda_tracks, overwrite=False)
sensor2_mm.add_data(tracks=gmlcc_tracks, overwrite=False)
meas_fusion_mm.add_data(tracks=meas_fusion_tracks, overwrite=False)
track_fusion_mm.add_data(tracks=track_fusion_tracks, overwrite=False)
```

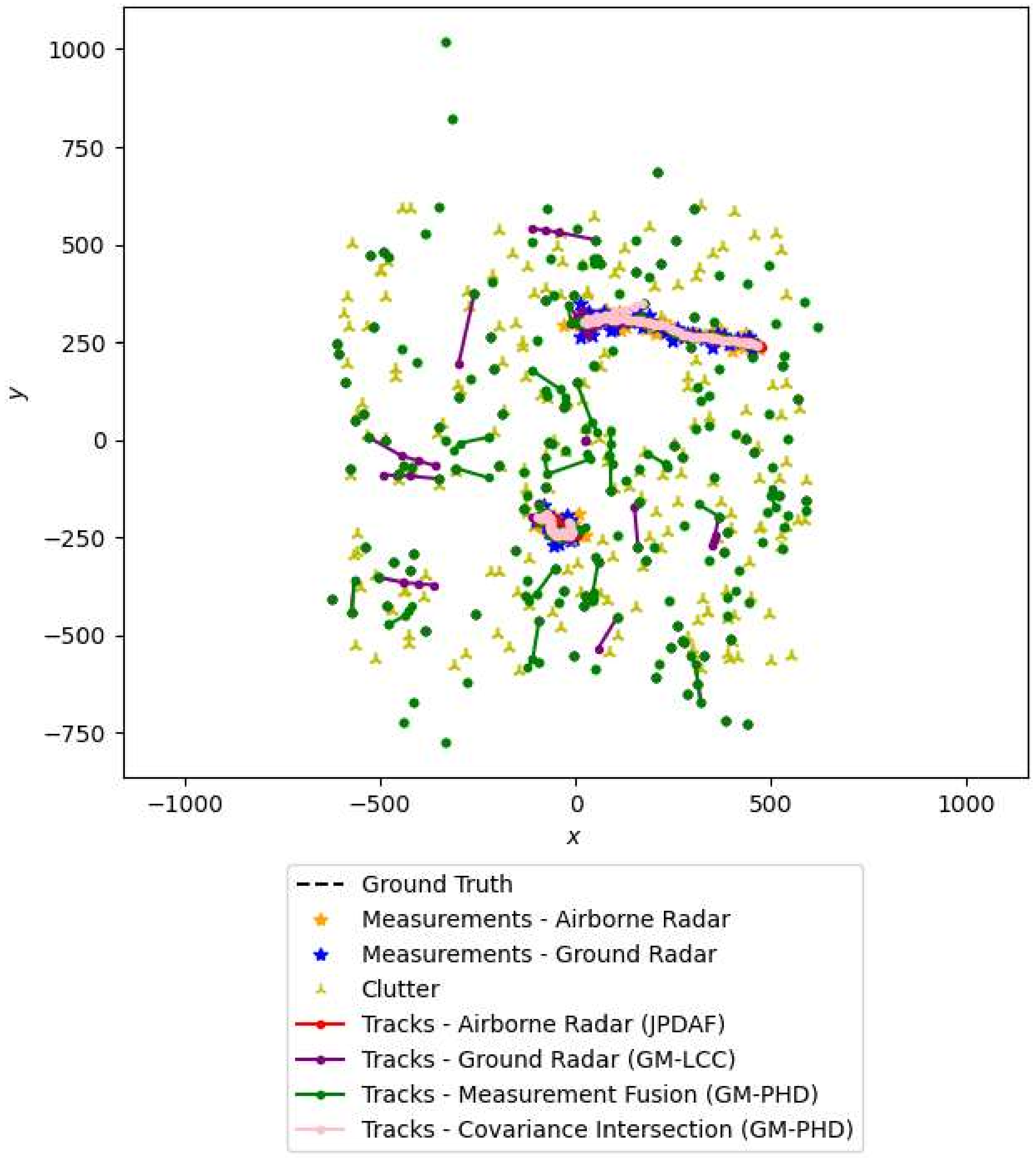
10: Plot the Results

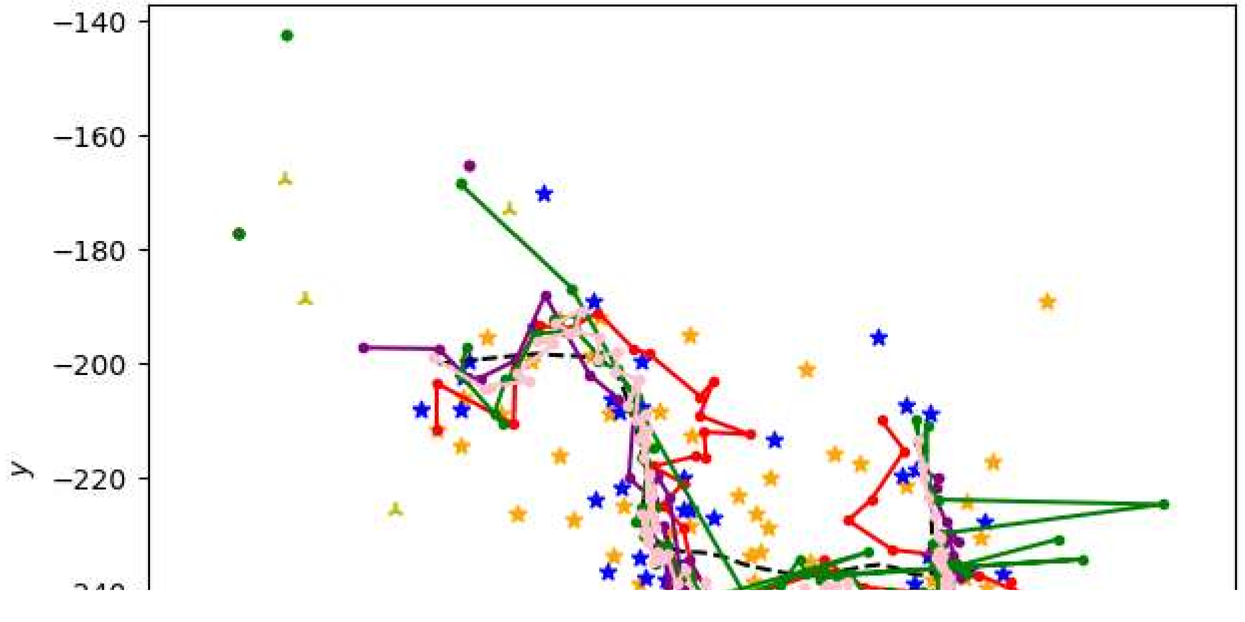
Next, we will plot all of the resulting tracks and measurements. This will be done in two plots. The first plot will show all of the data, and the second plot will show a closer view of one resultant track.

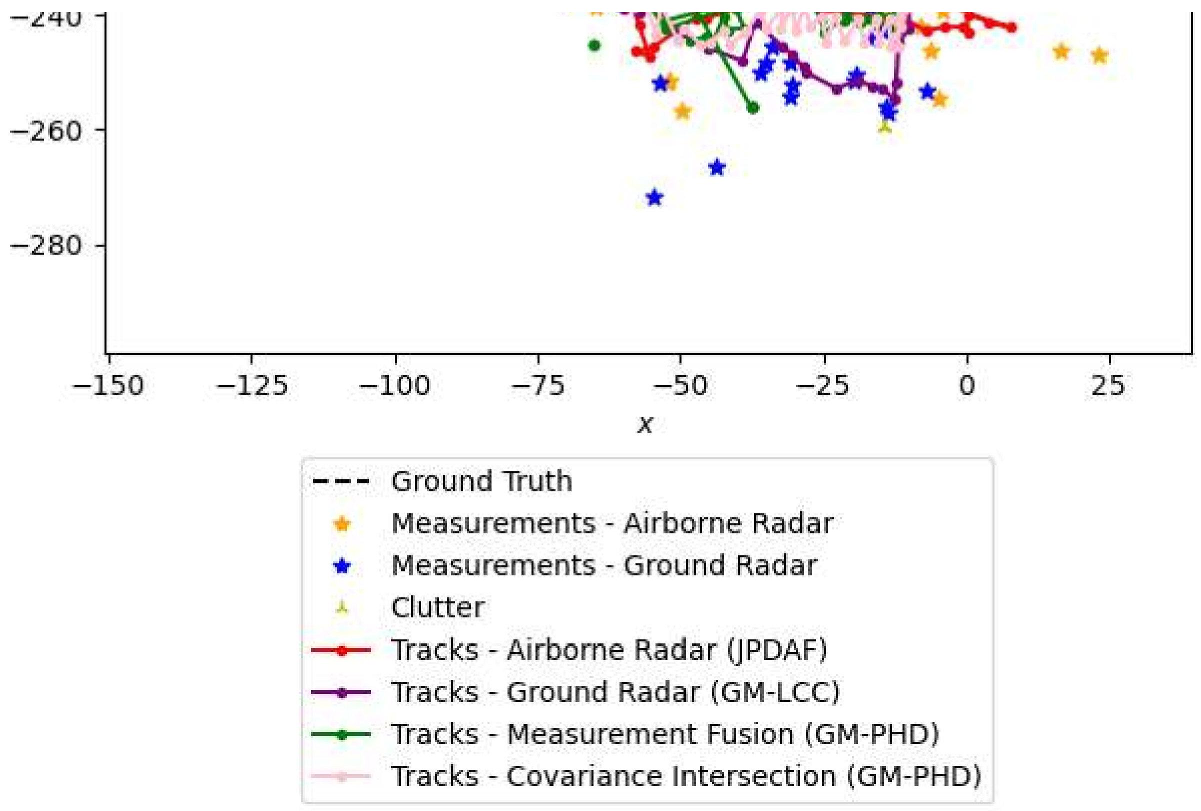
These plots are done in 2D to make them more readable. They are still very busy, so you are invited to run it yourself and remove some of the items from the plot. Or, if you are running the notebook locally, add the next line to your code to make it an interactive plot.

%matplotlib widget

```
plotter1, plotter2 = Plotter(), Plotter()
for plotter in [plotter1, plotter2]:
    plotter.plot_ground_truths(set(radar_simulator.groundtruth.groundtruth_paths), [0, 2],
                               color='black')
    plotter.plot_measurements(sensor1_detections, [0, 2], color='orange', marker='*',
                              measurements_label='Measurements - Airborne Radar')
    plotter.plot_measurements(sensor2_detections, [0, 2], color='blue', marker='*',
                              measurements_label='Measurements - Ground Radar')
    plotter.plot_tracks(jpda_tracks, [0, 2], color='red',
                        track_label='Tracks - Airborne Radar (JPDAF)')
    plotter.plot_tracks(gmlcc_tracks, [0, 2], color='purple',
                        track_label='Tracks - Ground Radar (GM-LCC)')
    plotter.plot_tracks(meas_fusion_tracks, [0, 2], color='green',
                        track_label='Tracks - Measurement Fusion (GM-PHD)')
    plotter.plot_tracks(track_fusion_tracks, [0, 2], color='pink',
                        track_label='Tracks - Covariance Intersection (GM-PHD)')

    # Format the legend a bit. Set the position outside of the plot, and
    # swap the order of the clutter and ground radar measurements
    pos = plotter.ax.get_position()
    plotter.ax.set_position([pos.x0, pos.y0, pos.width * 0.7, pos.height])
    k = list(plotter.legend_dict.keys())
    k[2], k[3] = k[3], k[2]
    v = list(plotter.legend_dict.values())
    v[2], v[3] = v[3], v[2]
    plotter.ax.legend(handles=v, labels=k, loc='lower center', bbox_to_anchor=(0.5, -0.5))

plotter1.fig.show()

track = track_fusion_tracks.pop()
x_min = min([state.state_vector[0] for state in track])
x_max = max([state.state_vector[0] for state in track])
y_min = min([state.state_vector[2] for state in track])
y_max = max([state.state_vector[2] for state in track])

plotter2.ax.set_xlim(x_min-50, x_max+50)
plotter2.ax.set_ylim(y_min-50, y_max+50)

plotter2.fig.show()
```


1000
750
500
250
0
−250
−500
−750
y
−1000
−500
0
500
1000
x
Ground Truth
Measurements - Airborne Radar
Measurements - Ground Radar
Clutter
Tracks - Airborne Radar (JPDAF)
Tracks - Ground Radar (GM-LCC)
Tracks - Measurement Fusion (GM-PHD)
Tracks - Covariance Intersection (GM-PHD)


−140
−160
−180
−200
−220
y

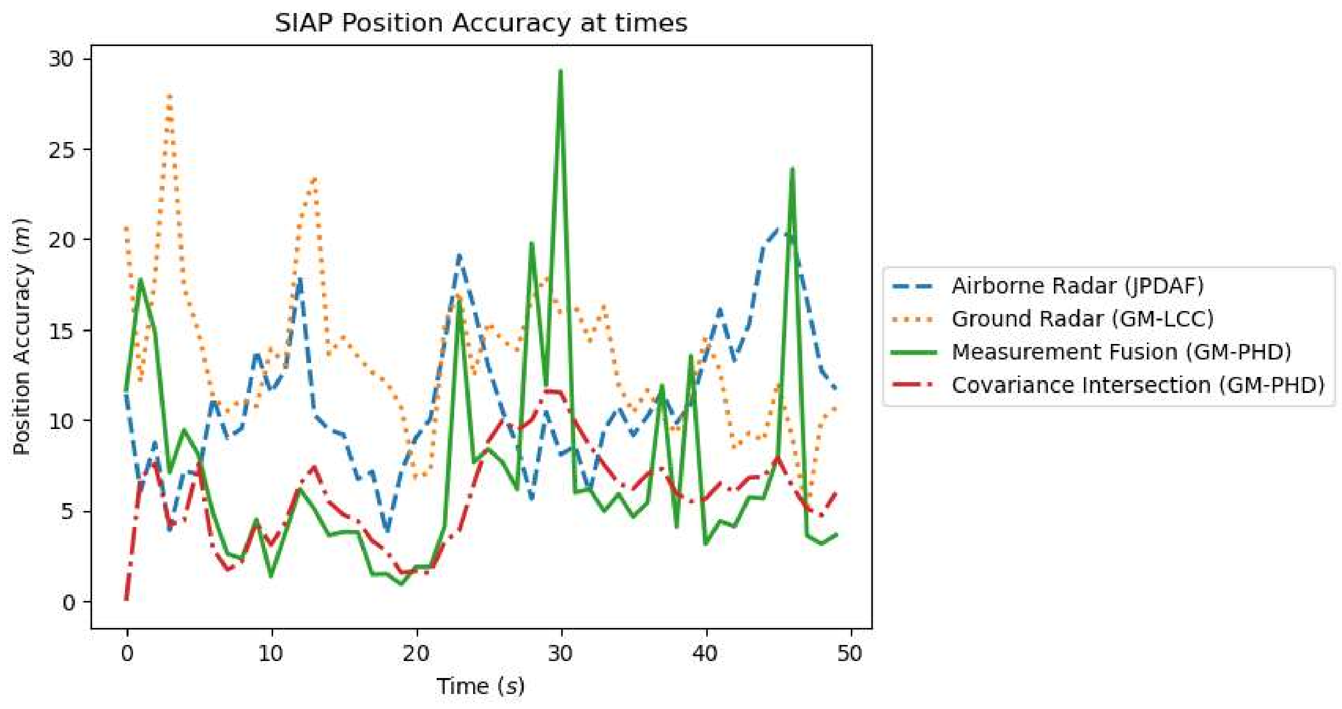


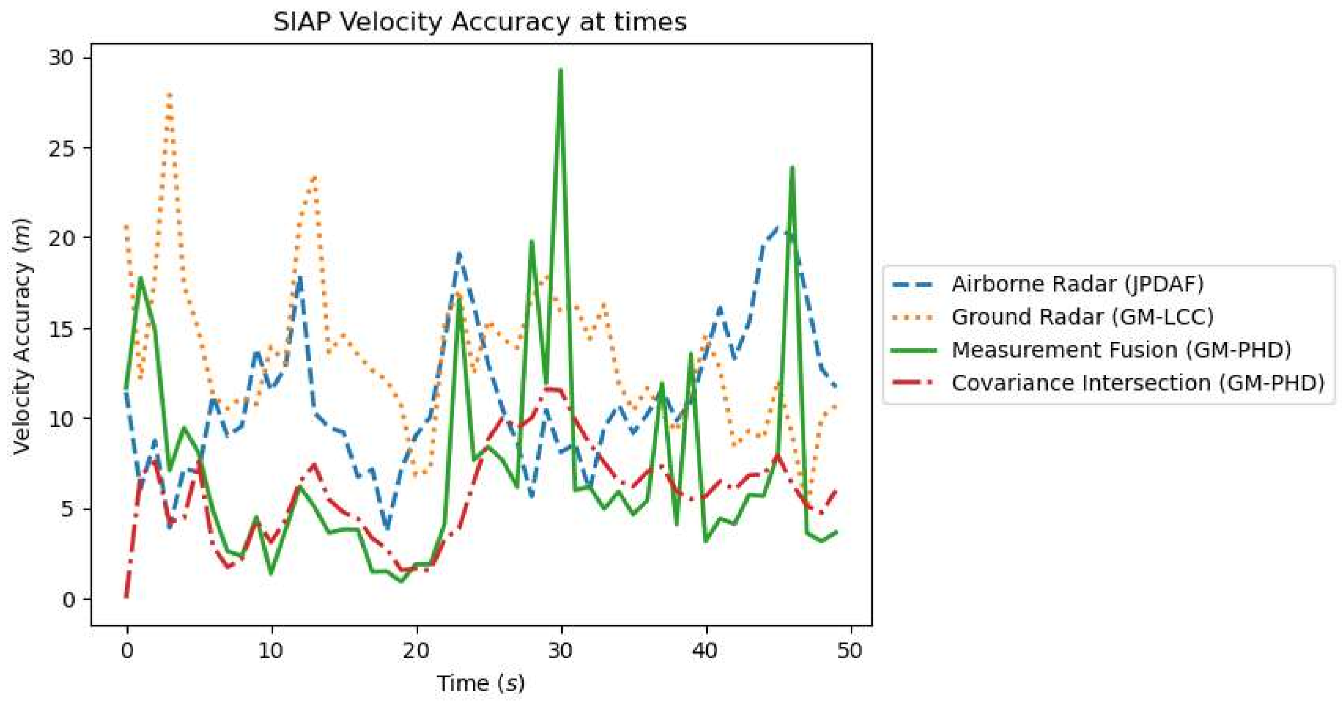
Now we will plot the metrics. First, we call a function for each sensor manager to calculate the metrics.

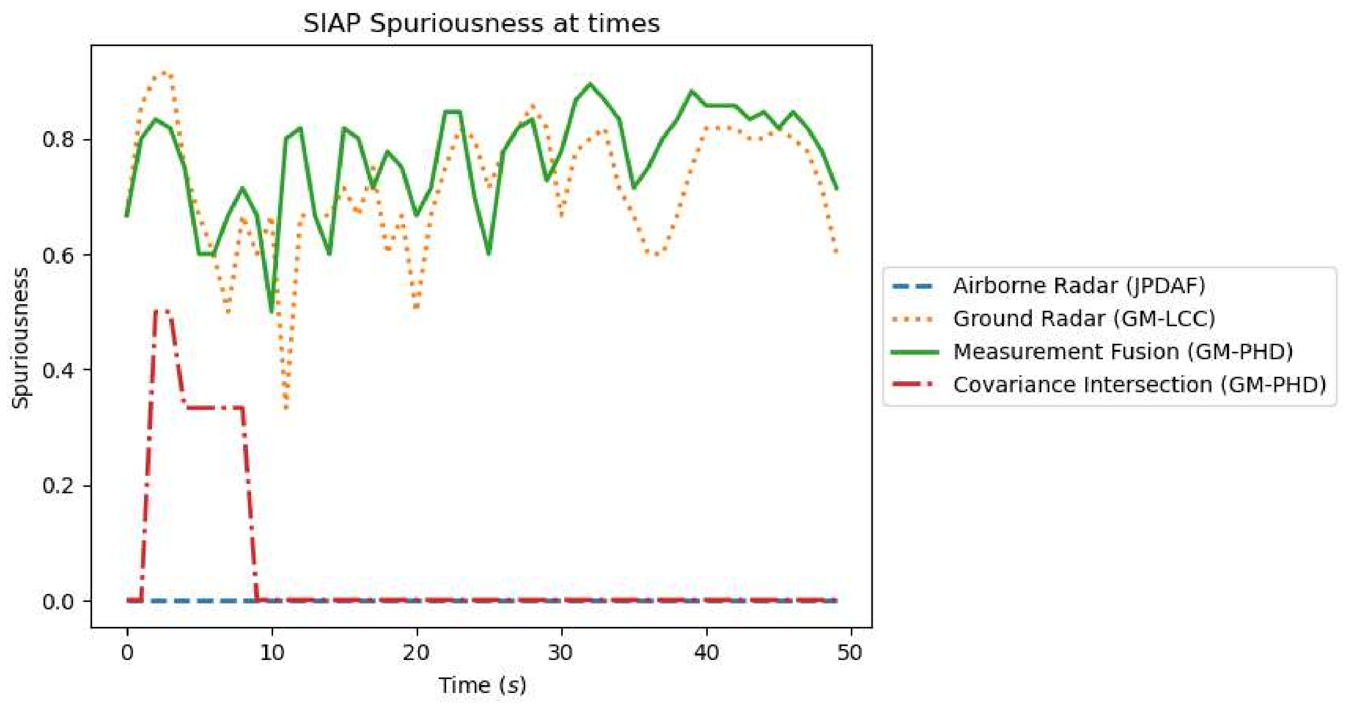
```
s1_metrics = sensor1_mm.generate_metrics()
s2_metrics = sensor2_mm.generate_metrics()
meas_fusion_metrics = meas_fusion_mm.generate_metrics()
track_fusion_metrics = track_fusion_mm.generate_metrics()
```

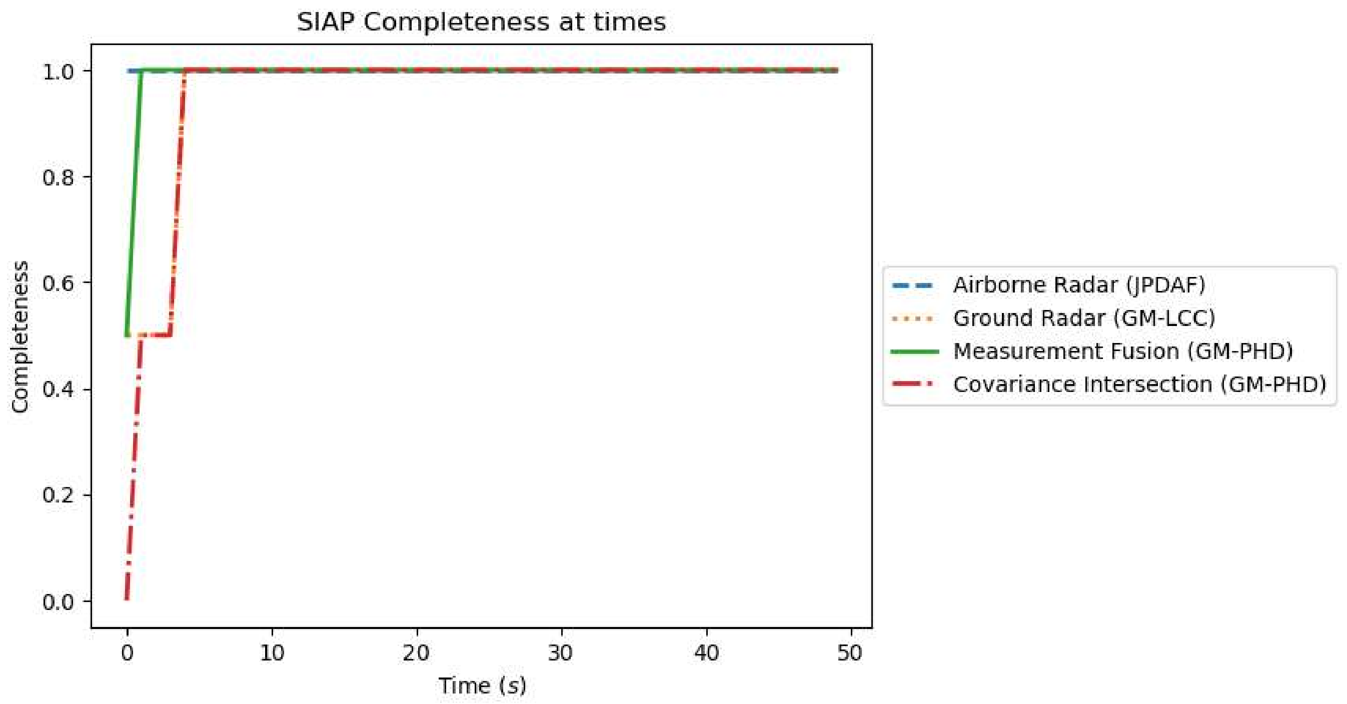
Now we can plot them. The SIAP and OSPA metrics can be done together in a loop. The Track-To-Truth ratio needs to be done separately so that it can be calculated at each timestep.

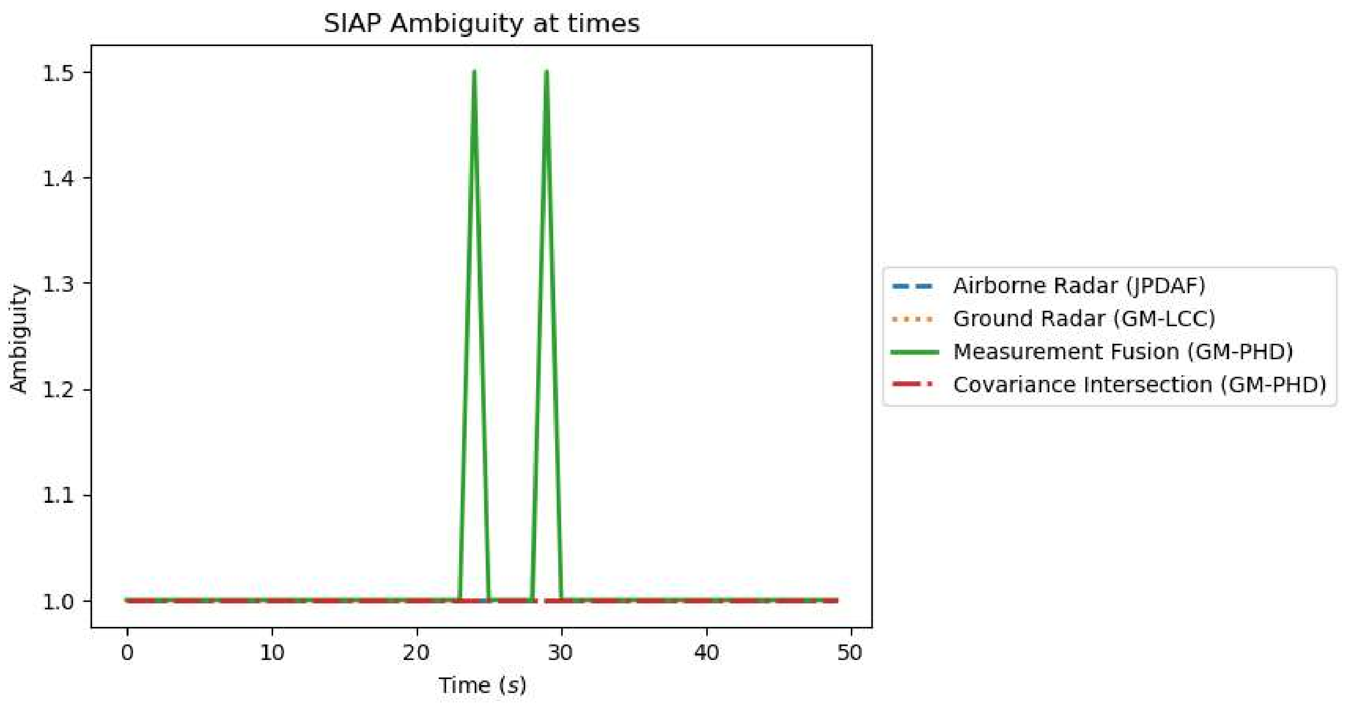
```python
from matplotlib import pyplot as plt
from stonesoup.metricgenerator.tracktotruthmetrics import SIAPMetrics

# Legend labels for each type of tracker
labels = ['Airborne Radar (JPDAF)', 'Ground Radar (GM-LCC)', 'Measurement Fusion (GM-PHD)',
'Covariance Intersection (GM-PHD)']
linestyles = ['dashed', 'dotted', 'solid', 'dashdot']

# Iterate through the SIAP and OSPA metrics
for metric_name in ['SIAP Position Accuracy at times', 'SIAP Velocity Accuracy at times',
                    'SIAP Spuriousness at times', 'SIAP Completeness at times',
                    'SIAP Ambiguity at times', 'OSPA distances', 'Sum of Covariance Norms
Metric']:
    fig, ax = plt.subplots()

    # Plot the metrics from each metric manager
    for tracker_metrics, label, line in zip([s1_metrics, s2_metrics, meas_fusion_metrics,
                                             track_fusion_metrics], labels, linestyles):
        metrics = tracker_metrics[metric_name]
        ax.plot([m.value for m in metrics.value], linewidth=2, label=label, linestyle=line)

    # Set x and y labels and title
    ax.set_xlabel("Time $(s)$")
    if metric_name.startswith('OSPA'):
        ax.set_title('OSPA Distance')
        ax.set_ylabel('Distance')
        ax.set_ylim(0, 12)  # change y axis range for OSPA distance
    elif metric_name.startswith('Sum of'):
        ax.set_title(metric_name)
        ax.set_ylabel('Sum of Covariance Norms')
    else:
        ax.set_title(metric_name)
        ax.set_ylabel(metric_name[5:-9])

    # Add units to y axis where applicable
    if metric_name.startswith('SIAP Position') or metric_name.startswith('SIAP Velocity') or
metric_name.startswith('OSPA'):
        ax.set_ylabel(ax.yaxis.get_label().get_text() + ' $(m)$')

    # Add legend
    ax.legend(loc='center left', bbox_to_anchor=(1.0, 0.5))


# Plot Track to Truth Ratio
fig, ax = plt.subplots()
times = sensor1_mm.list_timestamps()

# Iterate through the metric managers. For each one, go through the list of all timesteps
# and calculate the ratio at that time
for manager, label, line in zip([sensor1_mm, sensor2_mm, meas_fusion_mm, track_fusion_mm],
                                labels, linestyles):
    ratios = []
    for time in times:
        num_tracks = SIAPMetrics.num_tracks_at_time(manager=manager, timestamp=time)
        num_truths = SIAPMetrics.num_truths_at_time(manager=manager, timestamp=time)
        ratios.append(num_tracks / num_truths)
    plt.plot(ratios, linewidth=2, label=label, linestyle=line)

ax.set_title('Track-to-Truth Ratio')
ax.set_ylabel('Ratio')
ax.set_xlabel('Time $(s)$')
ax.legend(loc='center left', bbox_to_anchor=(1.0, 0.5))
```

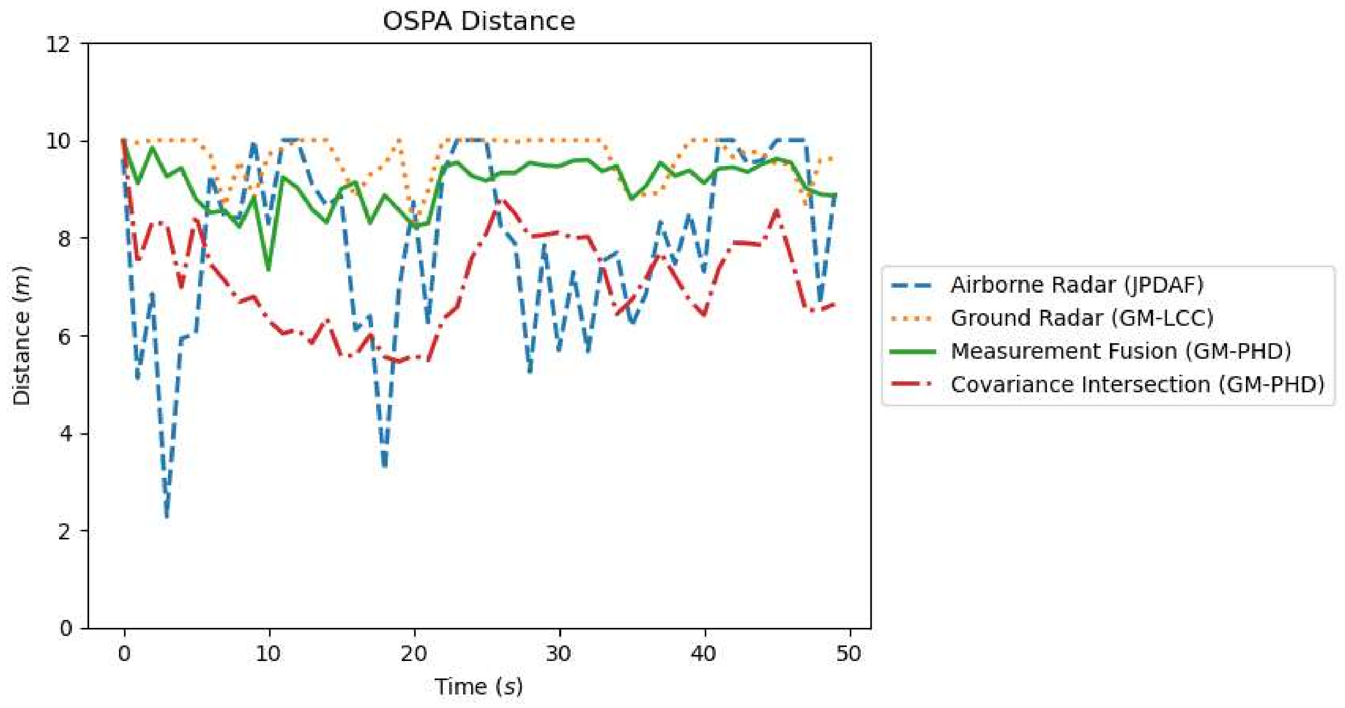


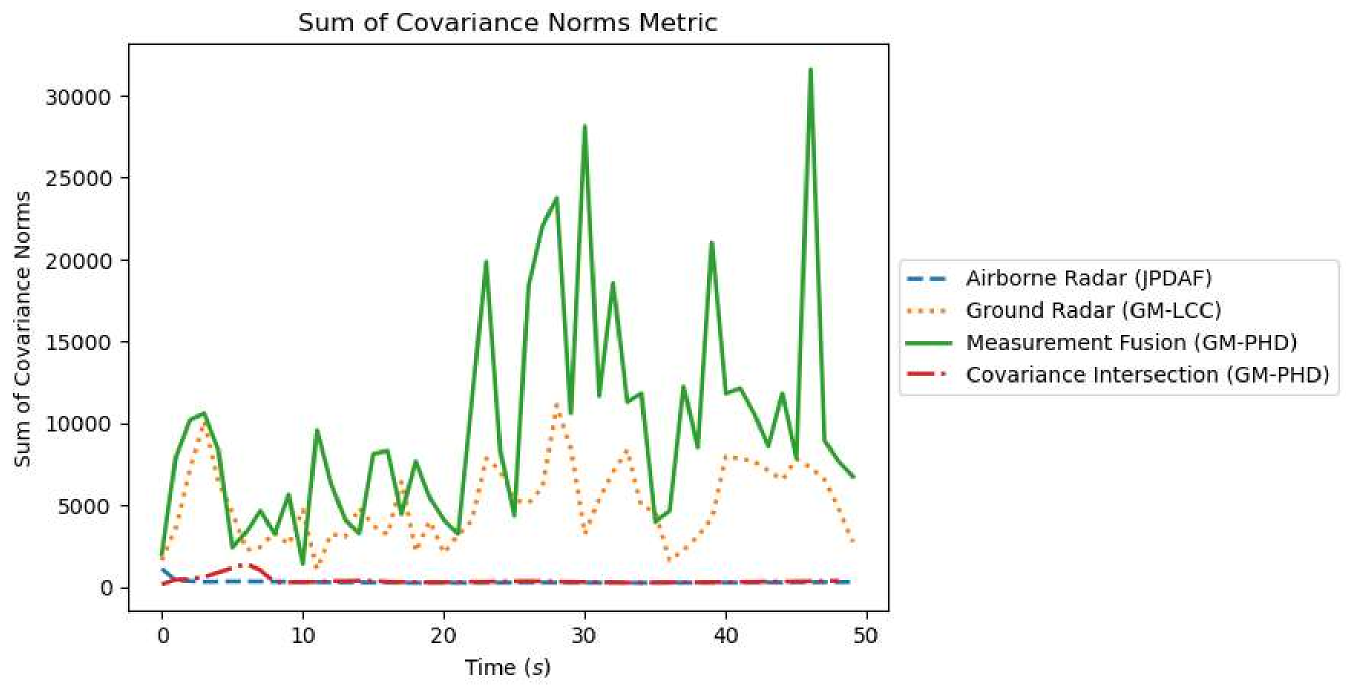


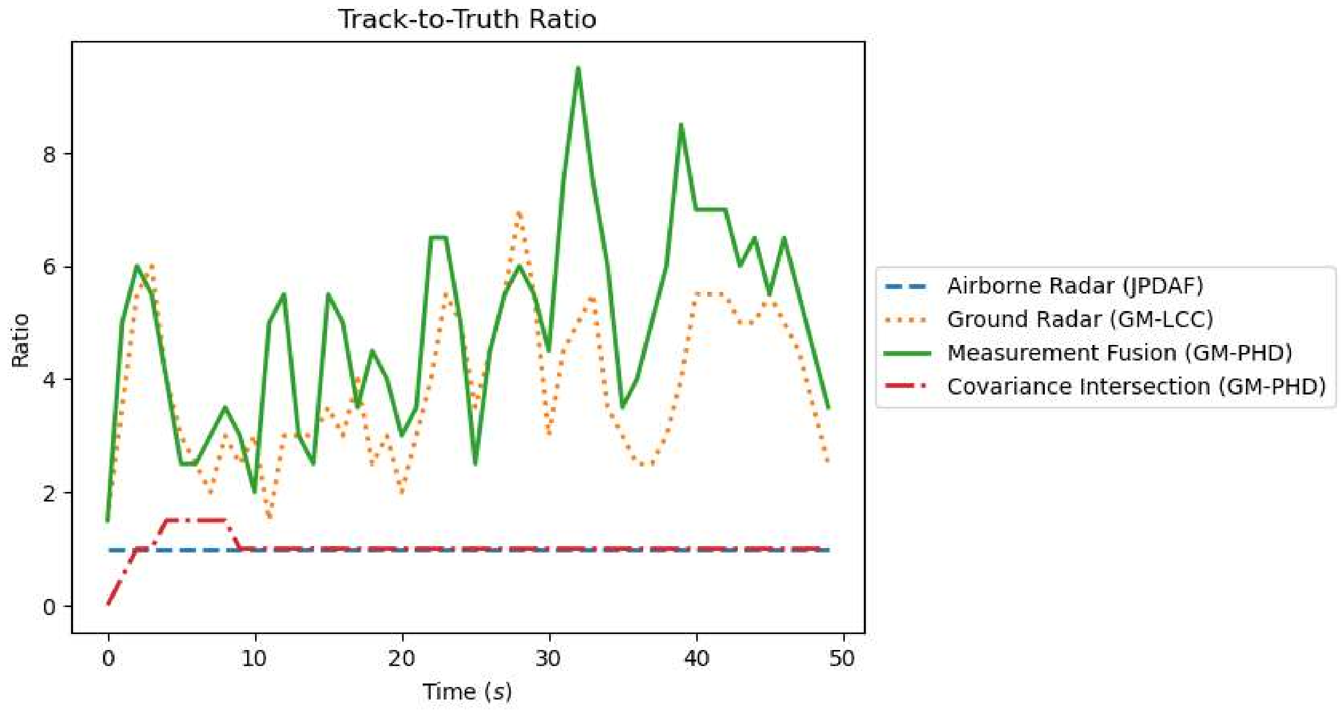

SIAP Spuriousness at times
Spuriousness
Time (s)
0.0
0.2
0.4
0.6
0.8
0
10
20
30
40
50
Airborne Radar (JPDAF)
Ground Radar (GM-LCC)
Measurement Fusion (GM-PHD)
Covariance Intersection (GM-PHD)


SIAP Completeness at times
Completeness
Time (s)
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
Airborne Radar (JPDAF)
Ground Radar (GM-LCC)
Measurement Fusion (GM-PHD)
Covariance Intersection (GM-PHD)


SIAP Ambiguity at times
Ambiguity
Time (s)
Airborne Radar (JPDAF)
Ground Radar (GM-LCC)
Measurement Fusion (GM-PHD)
Covariance Intersection (GM-PHD)


OSPA Distance
Distance (m)
Time (s)
Airborne Radar (JPDAF)
Ground Radar (GM-LCC)
Measurement Fusion (GM-PHD)
Covariance Intersection (GM-PHD)





References

[1] : Julier, S. J. and Uhlmann, J. K., “General decentralized data fusion with covariance intersection,” Handbook of multisensor data fusion: theory and practice, pp. 319–344, 2009.

[2] : Clark, D. E. and Campbell, M. A., “Integrating covariance intersection into Bayesian multi-target tracking filters,” preprint on TechRxiv. submitted to IEEE Transactions on Aerospace and Electronic Systems.

[3] : Hurley, M. B., “An information theoretic justification for covariance intersection and its generalization,” in Proceedings of the Fifth International Conference on Information Fusion.

FUSION 2002.(IEEE Cat. No. 02EX5997), vol. 1. IEEE, 2002, pp. 505–511

Total running time of the script: (0 minutes 11.013 seconds)

launch binder

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