PP Rework

Peter Gregory Husisian

May 2019

1 Introduction

Let $c_1, \ldots, c_n$ be represent a sequence of circles with corresponding locations $x_1, \ldots, x_n$ and times $t_1, \ldots, t_n$. Let $f_1, \ldots, f_n$ represent a sequence of cursor positions from a player logged at times $t_1, \ldots, t_n$. We seek the distribution:

$$\begin{aligned} p(f_1, \ldots, f_n | c_1, \ldots, c_n) &= p(f_1|c_1)p(f_2|f_1, c_1, c_2) \ldots p(f_{n-1}|f_1, \ldots, f_{n-2}, c_1, \ldots, c_{n-1})p(f_n|f_1, \ldots, f_{n-1}, c_1, \ldots, c_n) \\ &= \prod_{i=1}^{n} p(f_i|f_1, \ldots, f_{i-1}, c_1, \ldots, c_i) \end{aligned}$$

However, the increasing complexity of the distribution's factors make it intractable to both train and perform inference on even remotely long beatmaps. The Markov Assumption allows us to truncate the majority of each factor's conditional parameters, under the assumption that those events that occurred farther in the past play a lesser role when informing the probability distribution over the current element. In essence, one may construct a "kth order Markov Chain" by performing the following approximation:

$$p(f_1, \ldots, f_n | c_1, \ldots, c_n) \approx p(f_1|c_1)p(f_2|f_1, c_1, c_2) \ldots p(f_k|f_1, \ldots, f_{k-1}, c_1, \ldots, c_k) \prod_{i=k+1}^{n} p(f_i|f_{i-k}, \ldots, f_{i-1}, c_{i-k}, \ldots, c_i)$$

Whose factors conditionally rely on far fewer parameters, making inference significantly more feasible.

The sequence of cursor positions, $f_1, \ldots, f_n$, FC the song given by $c_1, \ldots, c_n$, if $\forall i, f_i \in R(c_i)$, where $R(c_i)$ is the set of all cursor positions that fall within hitcircle c_i (as determined by the CS of the map). Thus, the probability of FCing the map is the sum of the probabilities of all cursor sequences that would FC the map:

$$p(FC(c_1, \ldots, c_n)) = \sum_{[f_1, \ldots, f_n] \in FC(c_1,, c_n)} p(f_1, \ldots, f_n | c_1, \ldots, c_n)$$

Where $FC(c_1, \ldots, c_n)$ is the set of all possible cursor position sequences that would FC the map given by $c_1, \ldots, c_n$.

Under this simplification, we no longer need to concern ourselves with solving for the full joint distribution $p(f_1, \ldots, f_n | c_1, \ldots, c_n)$, but instead to optimize $p(f_i|f_{i-k}, \ldots, f_{i-1}, c_{i-k}, \ldots, c_i)$, which is significantly more feasible as it has far fewer parameters and far fewer possible input sequences. Because of this, a prohibitively large dataset of replay data would be no longer required, as we are concerning ourselves with a far simpler problem of "what's the probability of this note being hit given the k prior cursor positions and notes?" as opposed to "what's the probability of this hitcircle being hit given ALL prior cursor positions and notes" that the exact probability distribution would require.

Using this Markov Assumption framework, the complexity of the "probability to FC" framework would be completely governed by the complexity of the conditional probability approximation: $p(f_i|f_{i-k}, \ldots, f_{i-1}, c_{i-k}, \ldots, c_i)$. For example, one could opt to model this distribution using a Normal distribution whose parameters are given by some to-be-learned function, or choose a more complex distribution: perhaps a non-paramteric one, or one defined by a Neural Network.

Intuitively, the conditional distribution $p(f_i|f_{i-k}, \ldots, f_{i-1}, c_{i-k}, \ldots, c_i)$, from hereon simply referred to as $p(f_i| \ldots)$ is likely fairly simple. I would expect fairly good results even from simple distributions, such as the Normal distribution mentioned above.

As is, this approach is merely a proof of concept. It does not account for the movement to not sliderbreak, or for the fact that the player must click as well (although this is likely a simple inclusion in comparison to the distribution over cursor positions).